1.0 NUMERICAL PROCESSES (1) SURD

OBJECTIVES

By the end of this chapter you should be able to
* Distinguish between rational and non-rational numbers
* Identify number in surd form
* Simplify numerical surds
* Add, Subtract, Multiply & Divide surd
* Rationalized the denominator of fraction involving surd
* Multiply binomial surd
* Use conjugate to rationalize the denominator of surd with binomial fractions
* Express the trigonometrical ratio of 30°, 60°, 90° in term of surd.

1.1 SURD

Numbers such as 5, 2.1/3, 0.37, 0.6 √49 can be expressed as exact fractions or ratios 5/1, 7/3, 37/100, 2/3, ±7/1. These are called rational numbers, Numbers that can not be written as ratios are called irrational or non-rational numbers. π is an example of irrational number. π = 3.141592.

DISCUSSIONS/RULES

(1) √mn = √m x √n
(2) √m+n = √m + √n
(3) √m/n = √m / √n
(4) √m-n = √m - √n
(5) 2√m = √2m
(6) 3√n = √9n
(7) √x2y = x√y (Check example 3)
(8) √m x √m x √n = m√n
(9) m/√n = m/√n x √n/√n = m√n/n (/ mean divide all over)

1.2 SIMPLIFICATION OF SURD

Example 1. Simplfy;

(1) √45
= √9x5
= √9 x √5 (The square root of 9 is 3)
Ans = 3√5

(2) √162
= √81x2

√81 x √2 (The square root of 81 is 9)
Ans = 9√2

(3) √x2y
= √x2 x √y (x Raise to power 2 cut the √ of x)
Ans = x√y

(4) 3√50
= 3√25x2
= 3√25 x √2 (The square root of 25 is 5)
= 3x5 x √2
Ans = 15√2

Example 2. Express the ffg as square root of single number;

(1) 2√5
= √4 x √5 (The square root of 4 is 2)
= √4x5
Ans = √20

(2) 7√3
= √49 x √3 (The squre root of 49 is 7)
= √49x3
Ans = √147

1.3 ADDITION & SUBTRACTION SURD

Two ore more surd can be added together or subtracted from one another if they re likely surd. Before addition or subtraction the surds should first be simplified, if possible.

Example 3. Simplify the following

(1) 4√2 + 6√2
= (4+6)√2 i.e (4 & 6 is added because their √2 is the same)
Ans = 10√2

(2) 3√5 - 7√5
= (3-7)√5 i.e (3 is subtracted from 7 bcos their √5 is same)
Ans = -4√5

(3) 3√8 + √50
= 3√4x2 + √25x2

= (3√4 x √2) + (√25 x √2)
= (3x2 x √2) + (5 x √2) i.e (√4 is 2 & √25 is 5)
= 6 x √2 + 5 x √2
= 6√2 + 5√2
= (6+5)√2
Ans = 11√2

(4) 2√27 + √75 - 5√12
= 2√9x3 + √25x3 - 5√4x3
= (2√9 x √3) + (√25 x √3) - (5√4 x √3)
= (2x3 x √3) + (5 x √3) - (5x2 x √3)
i.e (√9 is 3, √25 is 5 & √4 is 2)
= 6√3 + 5√3 - 10√3
= (6+5-10)√3 (i.e their √3 is all the same)
= 1√3
Ans = √3

1.4 MULTIPLICATION SURD

When two or more surds are multiplied together, they should first be simplified. In division surd, if a faction has a surd in the denominator, it is usually best to rationalize the denominator.

Example. Simplify the following

(1) √27 x √50
= (√9x3) x (√25x2)
= (√9 x √3) x (√25 x √2)
= (3√3) x (5√2) i.e (Multiply both side)
Ans = 15√6

(2) √12 x 3√60 x √45
= (√4x3) x (3√4x15) x (√9x5)
= (√4 x √3) x (3√4 x √15) x (√9 x √5)
= (2√3) x (3x2√15) x (3√5)
= (2√3) x (6√15) x (3√5) i.e Open the bracket
= 2√3 x 6√15 x 3√5
= 2x6x3 √3x15x5
= 36√225 (square root of √225 is 15)
= 36 x 15
Ans = 540

(3) (2√5)^2 {^ means raise to power 2}
= 2√5 x 2√5

= 4 x 5 (Their root √5 is the same)
Ans = 20

OR

(2√5)^2 {^ means raise to power 2}
= (2)^2 x (√5)^2 i.e (2 raise to power 2 & the power 2 cut √ from √5)
= 4 x 5
Ans = 20 (Scroll up & check rules no7)

(4) √2 x √3 x √5 x √12 x √45 x √50
= √2x3x5x12x45x50
= √(2x50) x √(3x12) x √(5x45)
= √100 x √36 x √225
= 10 x 6 x 15
Ans = 900

(5) √3 x √6
= √3x6
= √18
= √9x2
= √9 x √2
Ans = 3√2

OR

√3 x √6
= √3 x √3x2
= √3 x √3 x √2 (Scroll up & check rules no8)
Ans = 3√2

1.5 DIVISION SURD

if a faction has a surd in the denominator, it is usually best to rationalize the denominator. To rationalize the denominator means to make the denominator into a rational number, usually a whole number. To do this multiply the numerator and denominator of the fraction by a surd that will make the denominator rational.

Example; Rationalize the denominator of the following.

(1) 6/√3
= 6/√3 x √3/√3
= 6 x √3 / √3 x √3 (Scroll up & check rules no9)
Ans = 6√3 / 3

(2) 7√18
= 7/√9 x √2
= 7/3√2
= 7/3√2 x √2/√2
= 7x√2 / 3x2 (Scroll up & check rules no9)
Ans = 7√2 / 6

(3) √18 / √2
= √18/2 i.e (18 divided by 2)
= √9
Ans = 3

(4) √5 / √2
= √5/2
Ans = √2.5 or 1/2√10

(5) √16 / √7
= 4 / √7
= 4/√7 x √7/√7 (Scroll up & check rules no9)
Ans = 4√7 / 7

(6) 5√7 x 2√3 / √45 x √21 (/ means all over)
= 5√7 x 2√3 / (√9x5) x (√3x7)
= 5√7 x 2√3 / (√9 x √5) x (√3 x √7)
= 5√7 x 2√3 / 3√5 x √3 x √7 (Cut both side, like √7 cut √7)
= 5 x 2 / 3√5
= 10 / 3√5
= 10/3√5 x √5/√5
= 10√5 / 3x5 (cut both side, 10 divided by 5)
Ans = 2√5 / 3

REVISION EXERCISE

Simplify the following;

[1] √20
[2] √75
[3] √150
[4] 3√10
[5] 2√11
[6] 3√2
[7] √5 x √10
[8] √6 x √8 x √10 x √12
[9] (2√3)^3
[10] 2/√2
[11] 6/√2
[12] 3√2 / √10
[13] √4 / √5
[14] (3√5 + 2)(√5 + 3)
[15] (4√3 + √2)(4√3 - √2)